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# METHODS IN COMPUTATIONAL PHYSICS

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# Computational Aspects of Numerical Models for Weather Prediction and Climate Simulation

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# United Kingdom Meteorological Office Five-Level General Circulation Model

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## I. Introduction

THE UNITED KINGDOM METEOROLOGICAL OFFICE general circulation model has been developed over a number of years. The first experiments to determine a suitable grid and finite difference systems were reported by Grimmer and Shaw (1967). The first version of the model was described by Corby *et al.* (1972), and the first long integration by Gilchrist *et al.* (1973). At that time, the grid, which was hemispheric only, was of regular latitude-longitude form with a spacing of  $3^\circ \times 5^\circ$ . This is approximately square in middle latitudes. To avoid very short time steps, the area from the pole to  $81^\circ\text{N}$  was treated as a polar cap so that only mean values of variables over the cap had to be carried. From  $81^\circ\text{N}$  to the latitude at which the longitudinal and latitudinal grid lengths were equal, a spatially variable time step, chosen so that the Courant-Friedrichs-Lewy condition for linear computational stability was satisfied locally, was used. When the model was reprogrammed for another computer, the grid and finite differences were changed to the second system tested by Grimmer and Shaw—namely, one



# A Description of the NCAR Global Circulation Models

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## I. Origin and Development of the NCAR Global Circulation Models

THIS ARTICLE DETAILS THE various National Center for Atmospheric Research (NCAR) global circulation models (GCM), showing their development for climate simulation and short-range weather forecasting. When modeling began in 1964 at NCAR, experience elsewhere with global or hemispheric primitive equation models was limited (see Kasahara in this volume). For example, in 1964, work on such models in the United States was progressing at the Geophysical Fluid Dynamics Laboratory (GFDL) (Smagorinsky *et al.*, 1965; Manabe *et al.*, 1965), Lawrence Radiation Laboratory (LRL) (Leith, 1965), and the University of California, Los Angeles (UCLA) (Mintz, 1964; Arakawa, 1966).

The GFDL and UCLA models used variants of the sigma vertical coordinate system devised by Phillips (1957) for incorporating orography. The LRL model used pressure as the vertical coordinate with no attempt to include orography. It was apparent that there were difficulties with the

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# Computational Design of the Basic Dynamical Processes of the UCLA General Circulation Model

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# Global Modeling of Atmospheric Flow by Spectral Methods

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